AP CALCULUS AB AND BC

UNIT 7

Differential Equations

AP EXAM WEIGHTING	6–12% AB 6–9% BC
CLASS PERIODS	~8–9 AB ~9–10 BC

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topics and skills.

Personal Progress Check 7

Multiple-choice:
~15 questions (AB)
~20 questions (BC)
Free-response: 3 questions

UNIT 7

AP EXAM WEIGHTING	6–12% AB	6–9% BC
CLASS PERIODS	~8–9 AB	~9–10 BC

Differential Equations

BIG IDEA 3
Analysis of Functions FUN

- How can we derive a model for the number of computers, C, infected by a virus, given a model for how fast the computers are being infected, $\frac{dC}{dt}$, at a particular time?

Developing Understanding

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth BC ONLY, students build understanding of topics introduced in Algebra II and other courses.

Building the Mathematical Practices

1.E 2.C 3.G 4.D

In this unit, students will translate mathematical information from one representation to another by matching equations and slope fields, rewriting verbal statements as differential equations, and sketching slope fields that match their symbolic representations. Provide students with explicit guidance on how to select an appropriate graphing technique. As students practice Euler's method, encourage them to transfer skills using tangent line approximations, rather than simply memorizing an algorithm BC ONLY.

Because the problems in this unit model real-world scenarios, help students to develop proficiency in transferring the mathematical procedures they've learned in "x's and y's" to equivalent environments with variable names other than x, y, and t. Using differentiation to confirm that solutions to differential equations are accurate and appropriate also helps students to develop an understanding of what it means to say that an equation is a solution to a differential equation.

Preparing for the AP Exam

Students should practice setting up and solving contextual questions involving separable differential equations until the solution strategy becomes routine: separate variables, antidifferentiate both sides of the equation and add a constant of integration, use initial conditions to determine the constant of integration, and rearrange the resulting expression to complete the solution. Failure to separate variables or omitting the constant of integration severely limits the number of points a student can earn on the AP Exam. A common error in antidifferentiation is to assume that all differential equations involving fractions have logarithmic solutions, presumably because some do.

Students should learn to recognize the forms of differential equations resulting in exponential and logistic BC ONLY models. These may be used or interpreted without performing the derivation. Students should also be reminded that differential equations give us information about the derivative and may be used directly to find information about a slope or rate of change.

UNIT 7

Differential Equations

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~8–9 CLASS PERIODS (AB) ~9–10 CLASS PERIODS (BC)
FUN-7	**7.1 Modeling Situations with Differential Equations**	2.C Identify a re-expression of mathematical information presented in a given representation.	
	7.2 Verifying Solutions for Differential Equations	3.G Confirm that solutions are accurate and appropriate.	
	7.3 Sketching Slope Fields	2.C Identify a re-expression of mathematical information presented in a given representation.	
	7.4 Reasoning Using Slope Fields	4.D Use appropriate graphing techniques.	
	7.5 Approximating Solutions Using Euler's Method BC ONLY	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	7.6 Finding General Solutions Using Separation of Variables	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	7.8 Exponential Models with Differential Equations	3.G Confirm that solutions are accurate and appropriate.	
	7.9 Logistic Models with Differential Equations BC ONLY	3.F Explain the meaning of mathematical solutions in context.	
	Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 7. Review the results in class to identify and address any student misunderstandings.		

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate various instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 199 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	7.3	**Match Mine** Give student pairs a blank 3×3 game board and nine graphs of slope fields, each on a separate card. Some should be in terms of x only, some in terms of y only, and some in terms of x and y. Be sure to include at least one trigonometric function. Student A arranges the graphs on the grid without showing Student B and then describes the arrangement so Student B can try to match it on their own board.
2	7.6	**Numbered Heads Together** Have each student complete the same problem individually (e.g., $y' = 2xy^2$, $\frac{dy}{dx} = y^2 + 1$, or $3y\,dy = (x^2 + 1)dx$). Make sure to use a variety of notation in whatever problem you pick. Then have students compare answers and procedures within groups. Students fix any mistakes until they all agree on the same answer.
3	7.7 7.8	**Collaborative Poster** Assign each student a role within their group: • Separating the variables • Integrating both sides • Finding C • Writing the final particular solution Then distribute a free-response question to each group and have them work on their assigned roles to solve the problem together. Examples include the following: • **2002 Form B #5(b)** (not transcendental) • **2011 #5(c)** (transcendental) • **2012 #5(c)** (transcendental) • **2014 #6(c)** (transcendental)

SUGGESTED SKILL

Connecting Representations

2.C

Identify a re-expression of mathematical information presented in a given representation.

AVAILABLE RESOURCE

- Classroom Resource > **Differential Equations**

TOPIC 7.1
Modeling Situations with Differential Equations

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.A

Interpret verbal statements of problems as differential equations involving a derivative expression.

ESSENTIAL KNOWLEDGE

FUN-7.A.1

Differential equations relate a function of an independent variable and the function's derivatives.

TOPIC 7.2
Verifying Solutions for Differential Equations

SUGGESTED SKILL

Justification

3.G

Confirm that solutions are accurate and appropriate.

AVAILABLE RESOURCE

- Classroom Resource > Differential Equations

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.B

Verify solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.B.1

Derivatives can be used to verify that a function is a solution to a given differential equation.

FUN-7.B.2

There may be infinitely many general solutions to a differential equation.

SUGGESTED SKILL

Connecting Representations

2.C

Identify a re-expression of mathematical information presented in a given representation.

AVAILABLE RESOURCES

- Classroom Resource > **Slope Fields**
- Classroom Resource > **Differential Equations**

TOPIC 7.3
Sketching Slope Fields

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.C

Estimate solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.C.1

A slope field is a graphical representation of a differential equation on a finite set of points in the plane.

FUN-7.C.2

Slope fields provide information about the behavior of solutions to first-order differential equations.

TOPIC 7.4
Reasoning Using Slope Fields

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.C

Estimate solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.C.3

Solutions to differential equations are functions or families of functions.

SUGGESTED SKILL

Communication and Notation

4.D

Use appropriate graphing techniques.

AVAILABLE RESOURCES

- Classroom Resource > **Slope Fields**
- Classroom Resource > **Differential Equations**

SUGGESTED SKILL

Implementing Mathematical Processes

1.E

Apply appropriate mathematical rules or procedures, with and without technology.

AVAILABLE RESOURCES

- Classroom Resource > **Approximation**
- Classroom Resource > **Differential Equations**

TOPIC 7.5
Approximating Solutions Using Euler's Method BC ONLY

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.C

Estimate solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.C.4

Euler's method provides a procedure for approximating a solution to a differential equation or a point on a solution curve. BC ONLY

TOPIC 7.6
Finding General Solutions Using Separation of Variables

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.D

Determine general solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.D.1

Some differential equations can be solved by separation of variables.

FUN-7.D.2

Antidifferentiation can be used to find general solutions to differential equations.

SUGGESTED SKILL

Implementing Mathematical Processes

1.E

Apply appropriate mathematical rules or procedures, with and without technology.

AVAILABLE RESOURCE

- Classroom Resource > Differential Equations

SUGGESTED SKILL

Implementing Mathematical Processes

1.E

Apply appropriate mathematical rules or procedures, with and without technology.

AVAILABLE RESOURCE

- Classroom Resource > **Differential Equations**

TOPIC 7.7
Finding Particular Solutions Using Initial Conditions and Separation of Variables

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.E

Determine particular solutions to differential equations.

ESSENTIAL KNOWLEDGE

FUN-7.E.1

A general solution may describe infinitely many solutions to a differential equation. There is only one particular solution passing through a given point.

FUN-7.E.2

The function F defined by $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$, satisfying $F(a) = y_0$.

FUN-7.E.3

Solutions to differential equations may be subject to domain restrictions.

TOPIC 7.8
Exponential Models with Differential Equations

SUGGESTED SKILL

Justification

3.G

Confirm that solutions are accurate and appropriate.

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
FUN-7.F Interpret the meaning of a differential equation and its variables in context.	**FUN-7.F.1** Specific applications of finding general and particular solutions to differential equations include motion along a line and exponential growth and decay. **FUN-7.F.2** The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.
FUN-7.G Determine general and particular solutions for problems involving differential equations in context.	**FUN-7.G.1** The exponential growth and decay model, $\frac{dy}{dt} = ky$, with initial condition $y = y_0$ when $t = 0$, has solutions of the form $y = y_0 e^{kt}$.

SUGGESTED SKILL

Justification

3.F

Explain the meaning of mathematical solutions in context.

TOPIC 7.9
Logistic Models with Differential Equations BC ONLY

Required Course Content

ENDURING UNDERSTANDING

FUN-7

Solving differential equations allows us to determine functions and develop models.

LEARNING OBJECTIVE

FUN-7.H

Interpret the meaning of the logistic growth model in context. **BC ONLY**

ESSENTIAL KNOWLEDGE

FUN-7.H.1

The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportional to the size of the quantity and the difference between the quantity and the carrying capacity" is $\frac{dy}{dt} = ky(a - y)$. **BC ONLY**

FUN-7.H.2

The logistic differential equation and initial conditions can be interpreted without solving the differential equation. **BC ONLY**

FUN-7.H.3

The limiting value (carrying capacity) of a logistic differential equation as the independent variable approaches infinity can be determined using the logistic growth model and initial conditions. **BC ONLY**

FUN-7.H.4

The value of the dependent variable in a logistic differential equation at the point when it is changing fastest can be determined using the logistic growth model and initial conditions. **BC ONLY**