AP CALCULUS AB AND BC

UNIT 4

Contextual Applications of Differentiation

AP EXAM WEIGHTING	10–15% AB 6–9% BC
CLASS PERIODS	~10–11 AB ~6–7 BC

Remember to go to **AP Classroom** to assign students the online **Personal Progress Check** for this unit.

Whether assigned as homework or completed in class, the **Personal Progress Check** provides each student with immediate feedback related to this unit's topics and skills.

Personal Progress Check 4

Multiple-choice: ~15 questions
Free-response: 3 questions

UNIT 4

AP EXAM WEIGHTING	10–15% AB	6–9% BC
CLASS PERIODS	~10–11 AB	~6–7 BC

Contextual Applications of Differentiation

BIG IDEA 1
Change CHA
- How are problems about position, velocity, and acceleration of a particle in motion over time structurally similar to problems about the volume of a rising balloon over an interval of heights, the population of London over the 14th century, or the metabolism of a dose of medicine over time?

BIG IDEA 2
Limits LIM
- Since certain indeterminate forms seem to actually approach a limit, how can we determine that limit, provided it exists?

Developing Understanding

Unit 4 begins by developing understanding of average and instantaneous rates of change in problems involving motion. The unit then identifies differentiation as a common underlying structure on which to build understanding of change in a variety of contexts. Students' understanding of units of measure often reinforces their understanding of contextual applications of differentiation. In problems involving related rates, identifying the independent variable common to related functions may help students to correctly apply the chain rule. When applying differentiation to determine limits of certain indeterminate forms using L'Hospital's rule, students must show that the rule applies.

Building the Mathematical Practices

1.D 1.F 2.A 3.F

Students will begin applying concepts from Units 2 and 3 to scenarios encountered in the world. Students often struggle to translate these verbal scenarios into the mathematical procedures necessary to answer the question. To solve these problems, students will need explicit instruction and intentional practice identifying key information, determining which procedure applies to the scenario presented (i.e., that "rates of change" indicate differentiation), stating what is changing and how, using correct units, and explaining what their answer means in the context of the scenario. Provide scenarios with different contexts but similar procedures so students begin to recognize and apply the reasoning behind those problem-solving decisions, rather than grasping at rules haphazardly.

Students must also be able to explain how an approximated value relates to the value it's intended to approximate. Students may not understand why they would use a tangent line approximation (i.e., linearization) rather than simply evaluating a function. Expose them to scenarios where an exact function value can't be calculated, and then ask them to determine whether a particular approximation is an overestimate or an underestimate of the function.

Preparing for the AP Exam

With contextual problems, emphasize careful reading for language such as, "find the rate of change." This will help students understand the underlying structure of the problem, answer the question asked, and interpret solutions in context. Students should not use words like "velocity" when they mean the rate of change in income, for example, even though the underlying structure is the same.

Emphasize that students must verify that $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ (or that both approach infinity) as a necessary first step before applying L'Hospital's Rule to determine $\lim_{x\to a} \frac{f(x)}{g(x)}$. Students should understand that $\frac{0}{0}$ or $\frac{\infty}{\infty}$ are appropriate labels for indeterminate forms but do not represent values in an equation. Therefore, it is incorrect to write $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$, for example. Note that $\lim_{x\to a} \frac{f(x)}{g(x)} \neq \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ when $\lim_{x\to a} g(x) = 0$. Also emphasize that the conclusion of L'Hospital's rule features the ratio of the derivatives of the numerator and denominator, respectively, rather than the derivative of the ratio.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~10–11 CLASS PERIODS (AB) ~6–7 CLASS PERIODS (BC)
CHA-3	**4.1 Interpreting the Meaning of the Derivative in Context**	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.	
	4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	4.3 Rates of Change in Applied Contexts Other Than Motion	2.A Identify common underlying structures in problems involving different contextual situations.	
	4.4 Introduction to Related Rates	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	4.5 Solving Related Rates Problems	3.F Explain the meaning of mathematical solutions in context.	
	4.6 Approximating Values of a Function Using Local Linearity and Linearization	1.F Explain how an approximated value relates to the actual value.	
LIM-4	**4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms**	3.D Apply an appropriate mathematical definition, theorem, or test.	
	Go to **AP Classroom** to assign the **Personal Progress Check** for Unit 4. Review the results in class to identify and address any student misunderstandings.		

SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional and are offered to provide possible ways to incorporate various instructional approaches into the classroom. Teachers do not need to use these activities or instructional approaches and are free to alter or edit them. The examples below were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit. Please refer to the Instructional Approaches section beginning on p. 199 for more examples of activities and strategies.

Activity	Topic	Sample Activity
1	4.1	**Quickwrite** Divide students into groups and give each group a context (outdoors, in a supermarket, in biology, in the government, at home, etc.). Students then write for a few minutes, listing things that are changing in that particular context.
2	4.2	**Create Representations** Provide verbal descriptions of a roller coaster ride: at time 0, velocity is 0 but about to become positive; at time 2, velocity is positive and increasing; at time 5, velocity is 0 and decreasing, etc. Have students graph position (from start), velocity, acceleration, speed, and then draw arrows at each point depicting whether their body would lean forward, backward, or not at all.
3	4.4	**Marking the Text** Have students read through a problem and highlight/underline the given quantities and directions in a problem, stating whether that information always applies or applies only at an instant.
4	4.5	**Round Table** Give students different related rates problems and a paper divided into five sections, titled as following: ▪ Draw a picture ▪ Equation ▪ Derivative ▪ Specific information used ▪ Interpretation Students first draw a picture of the situation and pass the papers clockwise. Students then critique the work in the previous section, complete the next section, and pass the papers again until all sections are completed.
5	4.6	**Scavenger Hunt** A starter question is posted in the room, for example, "Approximate the value . . . ." Have students work through the problem to find the value and then look for that value at the top of another card posted in the room. Students then solve the problem on that card, for example, "Write the equation of the tangent line . . . " and look for that solution on a third card, etc. The solution to the last problem will be on the starter card.

SUGGESTED SKILL

Implementing Mathematical Processes

1.D

Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.

AVAILABLE RESOURCE

- Professional Development > **Interpreting Context for Definite Integrals**

TOPIC 4.1
Interpreting the Meaning of the Derivative in Context

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.A

Interpret the meaning of a derivative in context.

ESSENTIAL KNOWLEDGE

CHA-3.A.1

The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.

CHA-3.A.2

The derivative can be used to express information about rates of change in applied contexts.

CHA-3.A.3

The unit for $f'(x)$ is the unit for f divided by the unit for x.

TOPIC 4.2
Straight-Line Motion: Connecting Position, Velocity, and Acceleration

SUGGESTED SKILL

Implementing Mathematical Processes

1.E

Apply appropriate mathematical rules or procedures, with and without technology.

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.B

Calculate rates of change in applied contexts.

ESSENTIAL KNOWLEDGE

CHA-3.B.1

The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.

SUGGESTED SKILL

Connecting Representations

2.A

Identify common underlying structures in problems involving different contextual situations.

TOPIC 4.3
Rates of Change in Applied Contexts Other Than Motion

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.C

Interpret rates of change in applied contexts.

ESSENTIAL KNOWLEDGE

CHA-3.C.1

The derivative can be used to solve problems involving rates of change in applied contexts.

TOPIC 4.4
Introduction to Related Rates

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.D

Calculate related rates in applied contexts.

ESSENTIAL KNOWLEDGE

CHA-3.D.1

The chain rule is the basis for differentiating variables in a related rates problem with respect to the same independent variable.

CHA-3.D.2

Other differentiation rules, such as the product rule and the quotient rule, may also be necessary to differentiate all variables with respect to the same independent variable.

SUGGESTED SKILL

Implementing Mathematical Processes

1.E

Apply appropriate mathematical rules or procedures, with and without technology.

AVAILABLE RESOURCES

- Professional Development > **Related Rates: Analyzing Problems in Context**
- AP Online Teacher Community Discussion > **Related Rates in FRQ**

SUGGESTED SKILL

Justification

3.F

Explain the meaning of mathematical solutions in context.

AVAILABLE RESOURCES

- Professional Development > **Related Rates: Analyzing Problems in Context**
- AP Online Teacher Community Discussion > **Related Rates in FRQ**

TOPIC 4.5
Solving Related Rates Problems

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.E

Interpret related rates in applied contexts.

ESSENTIAL KNOWLEDGE

CHA-3.E.1

The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.

TOPIC 4.6
Approximating Values of a Function Using Local Linearity and Linearization

SUGGESTED SKILL

Implementing Mathematical Processes

1.F

Explain how an approximated value relates to the actual value.

Required Course Content

ENDURING UNDERSTANDING

CHA-3

Derivatives allow us to solve real-world problems involving rates of change.

LEARNING OBJECTIVE

CHA-3.F

Approximate a value on a curve using the equation of a tangent line.

ESSENTIAL KNOWLEDGE

CHA-3.F.1

The tangent line is the graph of a locally linear approximation of the function near the point of tangency.

CHA-3.F.2

For a tangent line approximation, the function's behavior near the point of tangency may determine whether a tangent line value is an underestimate or an overestimate of the corresponding function value.

SUGGESTED SKILL

Justification

3.D

Apply an appropriate mathematical definition, theorem, or test.

AVAILABLE RESOURCES

- AP Online Teacher Community Discussion > **L'Hospital's Rule**
- AP Online Teacher Community Discussion > **Possible Inconsistent Language**
- The Exam > **2018 Chief Reader Report, FRQ #5(d)**
- The Exam > **2018 Samples and Commentary, FRQ #5(d)**
- The Exam > **2018 Scoring Guidelines, FRQ #5(d)**

TOPIC 4.7
Using L'Hospital's Rule for Determining Limits of Indeterminate Forms

Required Course Content

ENDURING UNDERSTANDING

LIM-4

L'Hospital's Rule allows us to determine the limits of some indeterminate forms.

LEARNING OBJECTIVE

LIM-4.A

Determine limits of functions that result in indeterminate forms.

ESSENTIAL KNOWLEDGE

LIM-4.A.1

When the ratio of two functions tends to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ in the limit, such forms are said to be indeterminate.

EXCLUSION STATEMENT

There are many other indeterminate forms, such as $\infty - \infty$, for example, but these will not be assessed on either the AP Calculus AB or BC Exam. However, teachers may include these topics, if time permits.

LIM-4.A.2

Limits of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.